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The Weibull functional form for SEP event spectra

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Abstract. The evolution of the kinetic energy spectra of two Solar Energetic Particle (SEP) events has been investigated through the Shannon's differential entropy during the different phases of the selected events, as proposed by [1]. Data from LET and HET instruments onboard the STEREO spacecraft were used to cover a wide energy range from ~ 4 MeV to 100 MeV, as well as EPAM and ERNE data, on board the ACE and SOHO spacecraft, respectively, in the range 1.6 – 112 MeV. The spectral features were found to be consistent with the Weibull like shape, both during the main phase of the SEP events and over their whole duration. Comparison of results obtained for energetic particles accelerated at corotating interaction regions (CIRs) and transient-related interplanetary shocks are presented in the framework of shock acceleration.

1. Introduction

The processes of solar energetic particle (SEP) acceleration are thought to be associated with solar flares and/or with shocks driven by coronal mass ejections (CMEs) in the corona and interplanetary space. The relative contribution of flares and CME-driven shocks and the details of the involved acceleration processes are still under debate (e.g., [2-3]). Particle acceleration to high energies occurs in a part of the solar corona that is not yet accessible to in situ measurements and until now, remote sensing has not enabled to disentangle the different mechanisms at play during the acceleration process. Information on them can be obtained through the time evolution of the SEP energy spectra derived from the particle flux registered in the interplanetary space. These energy spectra can be representative of the source spectra provided that the processes of particle release and particle transport in the interplanetary medium do not distort their original shape. During impulsive SEP events, particles should propagate fast enough that energy changes can be neglected; hence, their spectrum should be representative of the source spectrum (e.g. [4]). On the other hand, during gradual SEP events, the particle spectrum should be related to the characteristics of the CME-driven shock, and it can be highly variable due also to propagation effects. In case of the hybrid events, the phenomena leading to SEP events might involve both flare and shock acceleration [3] or a variable shock geometry acting on different seed particle populations [2].

The common theory to explain gradual SEP events is the diffusive shock acceleration (e.g., [5]), which predicts a power law spectrum, the spectral index and shock compression ratio (regardless of species) being linearly related. It was proposed that the power law spectra should roll over at high energies due to increasing diffusion coefficient with energy [6]. Nevertheless, the double power law is often used to explain the observed SEP spectra [2], although there have been few attempts to explain why the spectral breaks result in such function. Recently, [7] found that the Weibull distribution is the



best fit, with respect to the previous models, for the energy spectrum both at the Earth's shock passage and for the main phase of the 4 April 2000 SEP event. These authors suggested that the Weibull distribution might be associated with shock acceleration, explaining it in terms of a stochastic multiplicative process.

High energy particles can also be produced by shock waves in the heliospheric environment, such as the energetic storm particle (ESP) events, related to the passage of a CME driven shock wave past the Earth, and those associated with the shocks bounding corotating interaction regions (CIRs), produced by the interaction between fast and slow solar wind streams (e.g., [8]). At low heliographic latitudes, such CIRs are typically bounded by forward and reverse waves on their leading and trailing edges, respectively, that steepen into shocks generally at heliocentric distances greater than 2 AU [9]. In situ observations have indicated that such corotating shocks can accelerate ions up to 20 MeV/nucleon in energy [10-12]. We stress that transients and corotating shocks are systems where shock acceleration hypothesis can be tested against in situ observations.

In order to get insight in the SEP acceleration phenomena, we analyzed the evolution of particle spectra during properly selected SEP events, including their ESP phase, through the Shannon entropy method as described in section 2. Moreover, we tested the Weibull type shape both to SEP events and CIR events in section 3 and 4, respectively. Results are compared and discussed in section 5, where conclusions are also drawn.

2. Selection of SEP events

2.1. Data used and method of analysis

In order to minimize the effects related to the particle transport in the interplanetary space, we selected the 21 March 2011 and 26 December 2001 SEP events, because they are isolated, preceded by almost undisturbed solar wind (no magnetic clouds/CMEs) and magnetically well connected with the solar source. Data used to study the former event are 1 minute averaged proton fluxes measured by the Low Energy Telescope (LET) and the High Energy Telescope (HET) on STEREO A in 23 energy channels in the range 4 – 100 MeV [13-14]. For the latter event we retrieved 5 minute proton measurements from both ERNE [15] aboard SOHO and EPAM [16] aboard ACE at L1 location to cover the energy range from 1.6 to 112 MeV for a total of 26 energy channels. For both events, data are averaged on 1 hour basis. A proper calibration procedure was applied by combining the ERNE channels from 1.8 MeV to 5.1 MeV to obtain the ACE equivalent energy channel 1.9 - 4.8 MeV. Then, the ACE fluxes were rescaled by about a factor 2, obtained by performing a linear regression between the two ERNE and ACE equivalent channels.

For each event we define the relative weight functions $p_i(E, t)$, for particles having energy E between E_i and E_{i+1} at time t and differential flux $f_i(E, t)$ in each energy channel i (from 1 to the number of channels N):

$$p_i(E, t) = \frac{f_i(E, t)\Delta E_i}{\sum_i f_i(E, t)\Delta E_i}$$

where $\Delta E_i = E_{i+1} - E_i$ is the channel energy bin.

The weight functions are greater than zero and normalized to unity. Then, we compute the Shannon entropy SE [17] on the basis of the normalized quantities $p_i(E, t)$ as follows:

$$SE = - \sum_i p_i(E, t) \ln(p_i(E, t))$$

which is not affected by particle density variations. Indeed, it is sensitive only to changes in the particle spectral shape [1]. In particular, an increase of the Shannon entropy corresponds to a flattening of the SEP spectrum and viceversa. The normalized Shannon entropy (Δ) can be also computed by dividing SE to its maximum value $\ln(N)$.

In order to understand the different processes leading to the SEP event, we studied how the spectrum evolves in its different phases, based on the concept of the SE, as it can be used as a proxy for the spectrum changes as discussed by [1]. We remark that, as the SE is able to detect a change in the

particle spectrum, it can be useful to identify changes in the particle acceleration mechanisms (such as the shock passage or a vanishing well connection to the shock nose). Nevertheless, the SE is not linked to the nature of such a mechanism, which has to be investigated by other quantities.

2.2. The 21 March 2011 SEP event

A solar eruption on the far side of the Sun (W132°) as viewed from Earth was observed on 21 March 2011. It produced an X-ray flare and a fast halo CME (speed >1341 km/s) at 02:24 UT [18], accompanied by Type II and Type IV radio sweeps, indicating the presence of a propagating interplanetary shock. The LET and HET instruments on board STEREO A (located at W88° with respect to Earth) recorded a sudden increase in the proton differential fluxes at energies between 4 and 100 MeV (see upper panel of Figure 1).

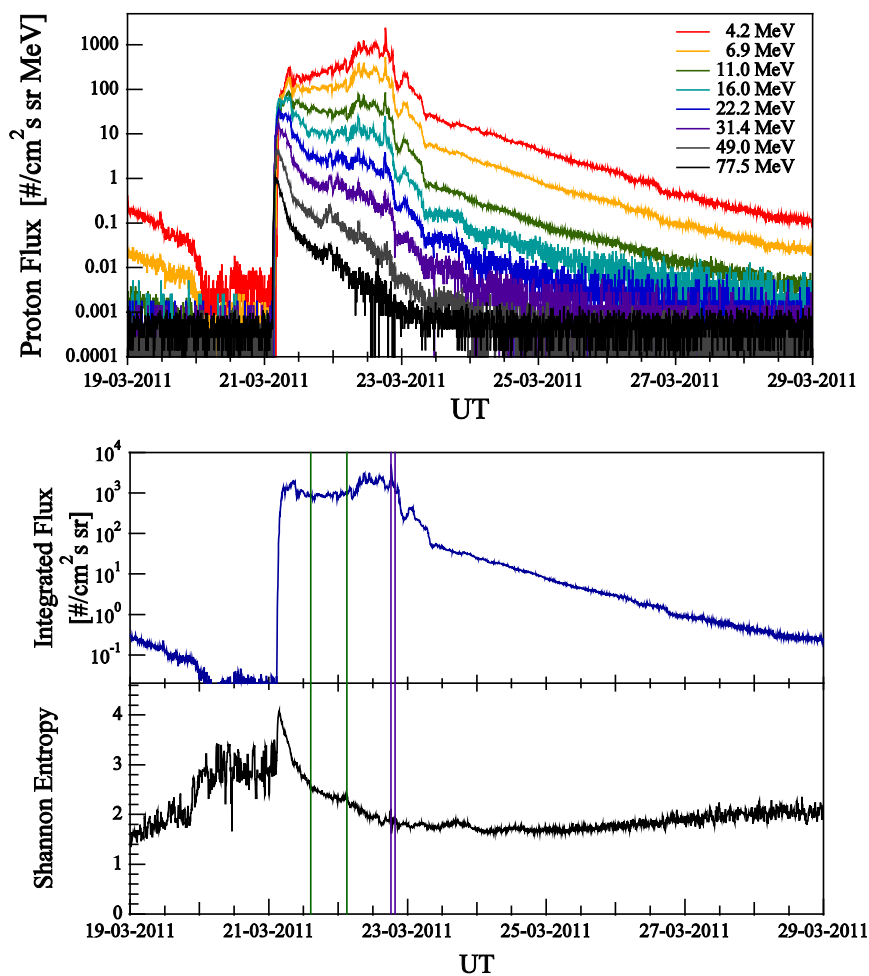


Figure 1. Proton differential flux (top) from STEREO A. The three lowest energy channels are from LET and the others from HET; not all channels are shown, but all are used in the spectra depicted in Figure 3. Energy integrated flux in the range 4–100 MeV (middle) and Shannon’s entropy (bottom) for the 21 March 2011 SEP event. Vertical lines indicate the time intervals during which the spectra shown in Figure 3 are derived. The shock passage is inside the period marked by the two violet lines.

The flux of high-energy protons (60–100 MeV) at STEREO A rises and falls over a 12 hr period but the flux of lower-energy protons rises steadily until $\sim 18:00$ UT on March 22 when an

interplanetary shock passes by STEREO A, with a discontinuous change in the solar wind speed from 420 to 770 km s⁻¹ [18]. The SEP peak flux is seen at the shock to form the ESP phase of the event. The SEP event can be classified as long-lasting gradual and was also observed at the Earth and L1 point, although much weaker than at STEREO A, which was more favourably located with respect to the associated solar flare (being the nominal azimuthal distance from the flare location less than 50° [19]). The > 10 MeV proton flux at geosynchronous orbit rose above 10 pfu on 21 March at 19:50 UT, reached a maximum of 14 pfu on 22 March at 01:30 UT and ended that day at 03:35 UT.

The lower panel of Figure 1 shows the time evolution of SE computed from STEREO A data. The SE time variation indicates that the SEP spectral shape continuously changes throughout the considered SEP event. In particular, a fast rise of the Shannon entropy can be observed during the prompt phase, because of the lack of low energy particles due to the velocity dispersion effect. Soon after, a relatively constant SE is observed, in a time interval (indicated by the vertical green lines in Figure 1) where also the energy integrated flux (middle panel of Figure 1) is almost flat. In this “plateau phase”, the spectrum can be considered to be almost stable.

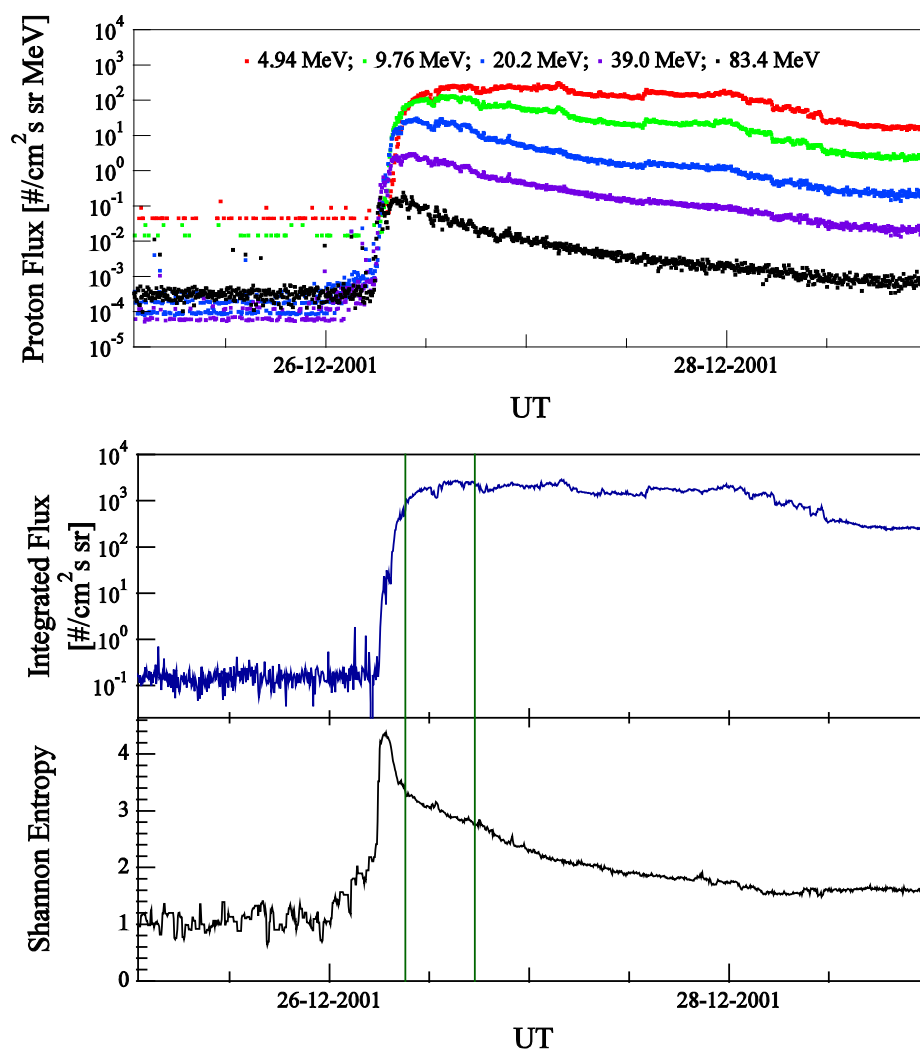


Figure 2. Proton differential flux (top) from SOHO/ERNE. Not all channels are shown, but all are used in the spectra depicted in Figure 4. Energy integrated flux in the range 1.6 -112 MeV (middle) and Shannon’s entropy (bottom) for the 26 December 2001 SEP event. Vertical green lines indicate the time during which the spectrum in Figure 4 is derived.

2.3. The 26 December 2001 SEP event

On 26 December 2001, the Sun released an M 7.6 X-ray flare, peaking at 05:36 UT, and a partial Halo CME (speed = 1446 km/s) at 05:30 UT (from the active region 9742, located at W54°), that were associated with a proton event, which extended to very high energies. It was recorded at geosynchronous orbit at energies greater than 100 MeV and 10 MeV. The greater than 100 MeV event began at 05:55 UT, reached a peak of 50 pfu at 07:20 UT, and ended at 19:40 UT. The greater than 10 MeV event began at 06:05 UT, reached a peak of 779 pfu at 11:15 UT, then ended at 10:40 UT on 28 December.

The 26 December 2001 SEP event can be classified as gradual and produced a ground level enhancement (numbered as GLE63), recorded by the worldwide network of neutron monitors. A sudden increase was measured also by the ERNE instrument in all the proton channels, without any electron contamination or saturation. The top panel of Figure 2 shows the flux profiles for some energy channels: the proton flux rise is fast and followed by a slower decay, which is still ongoing on 28 December. The energy integrated flux (middle panel of Figure 2) shows an almost flat profile after the prompt phase, while the SE (bottom panel of Figure 2), derived from ERNE data, gradually decreases after the first peak due to the velocity dispersion effect. The plateau phase is identified where SE changes less than 0.5 and it is delimited by the vertical green lines in Figure 2.

3. Model Fit

In order to study the energy content of the considered SEP events, we computed the particle spectrum both averaged over the whole duration of the event and in the plateau phase, where it is supposed to be relatively constant, such as the SE. This can be assumed as the main phase of the SEP event, that should also contain the main source acceleration processes imprint, although the transport processes may play a significant role that can modify the source acceleration spectrum. The main transport effects are supposed to be the following, as suggested by [20]: i) strong suppression of upstream ion intensities occurs at the plateau for energies near $\sim 1 \text{ MeV amu}^{-1}$, although GLEs with low intensities of 10–100 MeV protons on the plateau show spectra of ions that rise monotonically toward low energies with no peaking and no suppression of low-energy ions; ii) the temporal plateau formation does not necessarily imply a streaming limited spectrum (for instance the 1998 May 2 event studied by those authors has a reasonably well-defined temporal plateau even though strong streaming effects are absent). We tried to minimize these effects by using: 1) proton energies $> 1 \text{ MeV}$, in order to avoid the suppression at low energies; 2) medium size SEP events for which the wave amplification by the streaming protons is reduced; 3) events for which the spectra obtained for the temporal plateau phase do not show any plateau at lower energies; and 4) well-connected events. Moreover, considering that the spectra are not constrained by other factors such as scattering by ambient pre-event waves [20] and that the interplanetary conditions before the events were almost unperturbed, we can assume that the transport effects are reduced and that in the considered temporal plateau phase, the spectra still contains the imprint of the acceleration processes at the solar source.

For the 21 March 2011 SEP event, the spectrum was derived also for the ESP phase around the shock arrival, where particles are assumed to be locally accelerated by the shock, although the ESP phase may also contain particles trapped around the shock that were accelerated before the shock arrival.

Figure 3 depicts the spectra for the 21 March 2011 SEP event. As expected, the ESP phase of the event has a softer energy spectrum than the SEP plateau phase (left panel in Figure 3), as low energy particles are more easily accelerated at the shock passing the spacecraft location. Nevertheless, they can both be fitted with the same functional form, i.e., the Weibull like shape (also known as the two-parameter stretched exponential - see [21]) $W(E)$ defined as:

$$W(E) = kE^{b-1}e^{-\left(\frac{E}{E_0}\right)^b}$$

The Weibull functional form gives a good fit also to the spectrum derived for the whole duration of the SEP event (right panel of Figure 3). Similar results (see Figure 4) were obtained for the 26 December 2001 SEP event, in case of the whole event. In the plateau phase, consistency between the spectrum and the Weibull distribution is also found, because the data uncertainties for ERNE data are high. Nevertheless, at high energies the Weibull function departs from the actual spectrum, but we cannot rule out possible instrumental problems affecting the ERNE data.

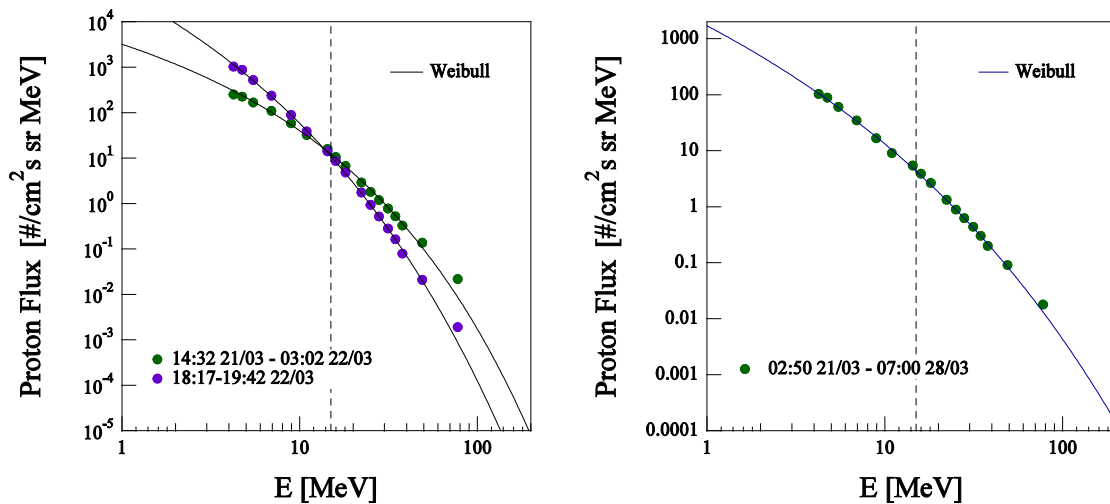


Figure 3. Time averaged spectrum over the plateau phase (green dots, left panel), at the shock arrival (violet dots, left panel) and over the whole duration of the event (right panel) obtained for the 21 March 2011 SEP event. Black curves are the Weibull function used to fit the spectra. The dashed line separates data from LET and HET instruments. Data errors are inside the marker size.

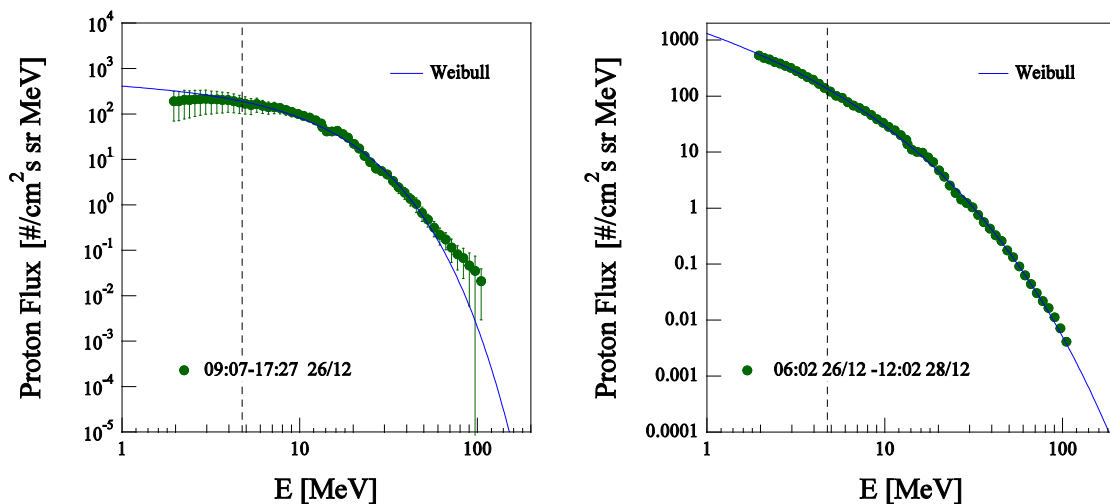


Figure 4. As in Figure 3 for the 26 December 2001 SEP event. Blue curves are the Weibull function used to fit the spectra. The dashed line separates data from ACE/EPAM and SOHO/ERNE instruments.

4. CIR events

We examined the temporal profiles and particle spectra for the 9 August 2008 and 8 January 2008 CIR events. A proton enhancement was recorded by the EPAM instrument onboard ACE in the energy range 0.047 – 4.75 MeV (upper left panel of Figure 5). The proton peak is found close to the compression region trailing edge (10 August, 03:46 UT), which was identified as a reverse shock [22]. The SE (lower left panel of Figure 5) suddenly decreases before the trailing edge, indicating a spectral

change due to the low energy population increase. The spectrum (right panel of Figure 5) was derived in the period (delimited by the vertical green lines in Figure 5) where SE varied slowly. Left upper panel of Figure 6 shows a CIR associated energetic proton enhancement at STEREO A, registered by the SEPT [23] and LET instruments in 34 energy channels from 0.10 – 12 MeV. A calibration procedure was applied by comparing the SEPT channel from 2.2 MeV to 6.5 MeV with the LET energy channel 4.0 - 4.5 MeV, because they have a comparable geometric mean energy (3.8 MeV and 4.2 MeV, respectively). Then, the LET fluxes were rescaled up by about a factor 5, obtained by performing a linear regression between data from the two channels. It can be seen that low energy (< 1 MeV) particle fluxes present two peaks, while the higher energy ones only one peak near the trailing edge (at 01:48 UT on 8 January) identified as a reverse shock [22]. Again, the SE (lower left panel) is slowly changing in the period around the shock passage (between the vertical green lines), where the spectrum is derived (right panel).

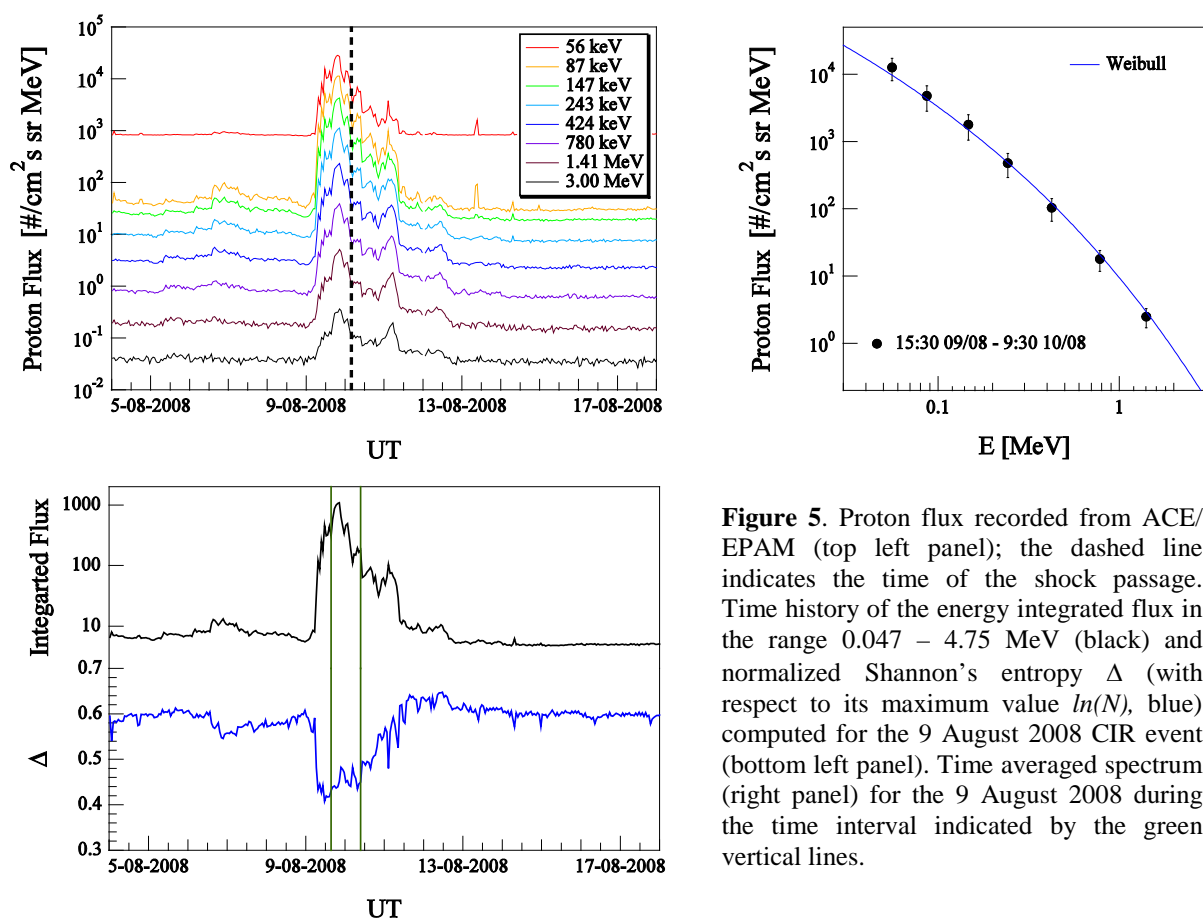


Figure 5. Proton flux recorded from ACE/EPAM (top left panel); the dashed line indicates the time of the shock passage. Time history of the energy integrated flux in the range 0.047 – 4.75 MeV (black) and normalized Shannon's entropy Δ (with respect to its maximum value $\ln(N)$, blue) computed for the 9 August 2008 CIR event (bottom left panel). Time averaged spectrum (right panel) for the 9 August 2008 during the time interval indicated by the green vertical lines.

5. Discussion and Conclusion

The SE has been used as a quantity able to describe changes in the SEP event spectral shape and identify the different phases of the event dynamics. For two SEP events we singled out a main phase, that we refer to as the plateau phase, which is not far from the prompt phase and for which we assume it may still contain the imprint of the acceleration at the solar source. Moreover, in the plateau phase the spectrum can be considered to be almost stable, given the constancy of the SE. In case of the 21 March 2011 SEP event, it was recognized an ESP phase, which is due to the particle acceleration in concomitance of the shock passage at STEREO A location. We applied the SE method also to two CIR events, for which a reverse shock was identified to exist at the trailing edge [22].

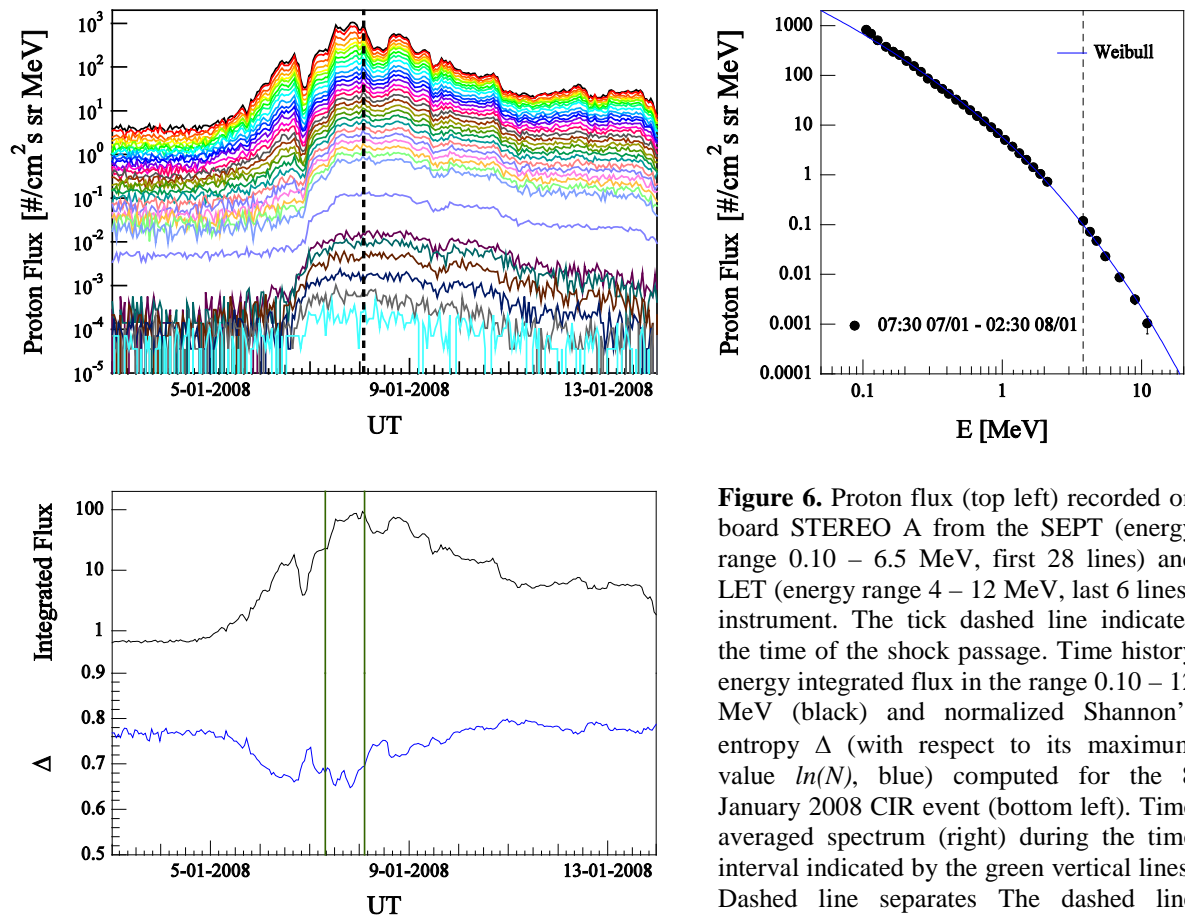


Figure 6. Proton flux (top left) recorded on board STEREO A from the SEPT (energy range 0.10 – 6.5 MeV, first 28 lines) and LET (energy range 4 – 12 MeV, last 6 lines) instrument. The tick dashed line indicates the time of the shock passage. Time history energy integrated flux in the range 0.10 – 12 MeV (black) and normalized Shannon's entropy Δ (with respect to its maximum value $\ln(N)$, blue) computed for the 8 January 2008 CIR event (bottom left). Time averaged spectrum (right) during the time interval indicated by the green vertical lines. Dashed line separates data from SEPT and LET.

Table 1. Fit parameters, by using the Weibull function, for the considered events.

Event	Event Phase	b	E_0 (MeV)
26 Dec. 2001 - SEP	Plateau phase	0.89 ± 0.04	6.1 ± 0.5
26 Dec. 2001 - SEP	Whole	0.46 ± 0.04	0.51 ± 0.02
21 Mar. 2011 - SEP	Plateau phase	0.450 ± 0.005	0.30 ± 0.02
21 Mar. 2011 - SEP	ESP	0.35 ± 0.02	0.02 ± 0.01
21 Mar. 2011 - SEP	Whole	0.30 ± 0.03	0.02 ± 0.01
9 Aug. 2008 - CIR	Reverse shock	0.30 ± 0.01	0.001 ± 0.0004
8 Jan. 2008 - CIR	Reverse shock	0.30 ± 0.01	0.002 ± 0.0004

A phase of slowly varying SE was identified behind the reverse shock, indicating that the spectrum is not greatly changing. Comparing the SE computed for CIR and SEP events an opposite trend is observed: the SE shows an abrupt increase in the SEP event prompt phase (due to the arrival first of the highest energy particles), whereas at the start of CIR events, the SE suddenly decreases due to the increase of the lowest energy population. We derived the spectra both in the identified phases and for the whole duration of the considered events. The same functional form (Weibull) was used to perform a fit, whose results are listed in Table 1, where b is the exponent and E_0 the rollover energy. In all cases the Weibull function is found to reproduce the observed energy spectra. This suggests that the acceleration mechanism at work could be the same for SEP, ESP and CIR events under study here.

The good consistency with the Weibull form in case of the ESP phase of the 21 March 2011 SEP event and of the reverse shocks in CIR events, confirms that shock acceleration observed in situ can be associated with the Weibull form, as proposed by [7]. This can be understood by considering that the first order Fermi acceleration, which is supposed to be effective at shock waves, is a stochastic multiplicative process. In the theory of large/extreme deviations, the Weibull shape is associated with the probability density function resulting from a multiplicative process of random variables [21], when assuming a threshold to mimic the system size limitation. Hence, the Weibull shape, which has not to be confused with a limited power law with an exponential roll-over, could provide a good description for particle acceleration occurring in a finite region and also resulting from a finite number of generations in the multiplicative process. In fact, the b parameter of the Weibull shape is inversely related to the number of generations involved in the multiplicative process [24]. Hence, the b parameter could be related to the permanence time in the acceleration region (or to the particle escape probability) as well as the shock strength. Thus, the proposed function might take into account important corrections to the classical diffusive shock acceleration scenario, from which a pure power-law is derived (assuming an infinite plane shock implying a huge number of generations in a multiplicative process).

The Weibull distribution shape is also able to reproduce the SEP spectrum in the plateau phase of SEP events, which supports a shock acceleration mechanism in the high corona. Nevertheless, in the GLE event an additional acceleration source cannot be excluded. In fact, during its main phase (where we assume the source acceleration imprint is still present) the Weibull function departs from the actual spectrum at high energies (although the fit is still good because of high error bars). This could be due to instrumental problems affecting the ERNE data or to a contribution from another acceleration source. On the other hand, in the case of 21 March 2011 SEP event, the Weibull provides a very good fit in all phases of the event (also even during the prompt phase). It seems not probable that flare and shock components are both present. In case of a flare contribution to the acceleration, i.e., if two accelerators are at work, spectra are expected to harden with increasing energy, as the flare should be more prolific than the shock at producing the highest energy particles. Hence, a break in the spectrum is expected at some energy as proposed by [4], instead of a function with sub-exponential decay such as the Weibull. According to [4]: “Of course, it is possible that the relative strengths and spectral shapes of the two components could be such that they minimize the appearance of a spectral break. But if the two components truly are independent, there is no reason to believe that such an arrangement should generally be the case. This spectral hardening should be particularly acute in event-integrated spectra, where the highest energy particles are produced by the flare for a brief time, while the shock continues to pump up the softer, lower end of the spectrum over an extended period.” This is not the case in the considered SEP events (something similar happens only in the plateau phase of the 26 December 2001 SEP event). Nevertheless, the good agreement between the spectrum derived over the whole SEP events and the Weibull function, suggests that both SEP events are dominated by the shock acceleration process.

Finally, we suggest that both b and E_0 parameters of the Weibull function could be related to the shock strength, and hence to the acceleration efficiency, as they are lower for the reverse shocks and ESP phase than for the plateau phase, where the coronal shock is supposed to be more intense. This can be supported by comparing the values of the Mach number obtained for the considered ESP and the 8 January CIR event (1.9 and 1.28, respectively; available at http://www-ssc.igpp.ucla.edu/~jlan/STEREO/Level3/STEREO_Level3_Shock.pdf) with their b values (0.35 ± 0.02 and 0.30 ± 0.01 , respectively). However, more work is needed to provide a theoretical evidence for the link between this spectral shape and particle acceleration mechanisms.

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<http://www.srl.caltech.edu/ACE/ASC/level2>; <http://www.srl.caltech.edu/STEREO>. Information about the solar sources of SEP events were obtained from <ftp://ftp.swpc.noaa.gov/pub/warehouse/>.

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